

WJEC (Wales) Physics A-level

Topic 3.3: Kinetic Theory

Notes

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Investigating Gases

The definition of a gas is a **state of a substance which expands to fill any volume** that it is placed in, setting it apart from other states like solids and liquids which don't do this.

Imagine a box containing just one particle which bounced around inside it. Given the initial position and velocity of the particle we could find its position and velocity at any future time. We could also then place a second particle into the box, and repeat the same method to find their position and velocities, but now with two particles it will get more complicated as they might collide with each other.

In fact even doing this by hand for say 3 particles would be a nightmare. An average gas contains around 10^{23} particles which makes calculating how the gas behaves by this method absolutely impossible, even for powerful computers.

Instead, physicists use **thermodynamic, measurable** quantities to describe the gas. These include quantities like pressure and temperature, which are defined below:

Pressure (p): The force experienced on one unit area of surface in contact with the gas.

Temperature (T): A measure of the average kinetic energy of the particles in the gas, measured in Kelvin (K)

Volume (V): A measure of the total space occupied by the gas, measured in m^3 .

Number of moles (n): A measure of the number of particles in the gas, measured in moles (6.02×10^{23} particles)

Experimentally it is quite easy to measure the first three of these quantities: Pressure, Temperature and Volume. Various physicists in the 19th century and before set up experiments to measure how these quantities were related to each other. They found the relations shown here:

Boyle's Law

When temperature is held constant, the pressure is inversely proportional to volume:

$$p \propto \frac{1}{V} \quad \text{at constant } T$$

Charles's Law

When pressure is constant, the volume is proportional to temperature:

$$V \propto T \quad \text{at constant } p$$





The Pressure Law

When volume is held constant, the pressure is proportional to temperature:

$$p \propto T \quad \text{at constant } V$$

Equation of State of an Ideal Gas

The three relationships between p , T and V shown above mean that all three must satisfy this:

$$pV \propto T$$

We haven't considered the **number of particles** in the gas yet, but if we imagine keeping V and T constant, and then double the number of particles in the system we would expect the pressure to double. This implies that:

$$pV \propto NT$$

With N as the **total number of particles**. Experimentally we can measure all of these quantities, allowing us to find the **constant of proportionality**. This constant is very important in thermal physics, it is called the **Boltzmann Constant** with a value:

$$k_B = 1.38 \times 10^{-23} \text{ m}^2\text{kg s}^{-2}\text{K}^{-1}$$

This gives us the **equation of state of an (ideal) gas**:

$$pV = k_B NT$$

N is usually not a very useful quantity to work with as it is usually very large. Instead we can express the number of particles in the gas in terms of the **number of moles of particles**.

One mole is defined to be the number of particles in 12g of Carbon-12. It is just a number with an incredibly large value, called **Avagadro's Number**:

$$N_A = 6.02 \times 10^{23} \text{ per mol}$$

Now we can express the total number of particles N as a multiple of this number, which we call **the number of moles, n** :

$$N = nN_A$$





Substituting this into our ideal gas equation gives:

$$pV = nN_A k_B T$$

Seeing as both N_A and k_B are constants, we can set a new constant $R = N_A k_B$, which is called the ideal gas constant, and has a value $R = 8.31 \text{ JK}^{-1} \text{ mol}^{-1}$

We then get a different form of the **equation of state**:

$$pV = nRT$$

This equation is also called the **Ideal Gas Equation**.

Moles and Relative Molecular Mass

The **relative molecular mass** M_r is the mass of a molecule, relative to $1/12^{\text{th}}$ that of Carbon-12. For example Hydrogen approximately has $M_r = 2$, meaning Hydrogen molecules are about $1/6$ the mass of one Carbon-12 atom.

We can also define the **molar mass** M_m , which is the mass per mole. From the definition of a mole, the molar mass of Carbon-12 must be $0.012 \text{ kg mol}^{-1}$. This indicates that M_m and M_r are related via:

$$M_m = \frac{M_r}{1000} \text{ kg}$$

We already know we can find the number of moles using $n = N/N_A$, multiplying the fraction through by m , the mass of one particle/molecule gives:

$$\text{number of moles} = n = \frac{mN}{mN_A} = \frac{\text{total mass}}{\text{molar mass}}$$

Assumptions of the Ideal Gas

It turns out that real gases do not perfectly satisfy the ideal gas equation. As an example, when a gas is squeezed into a smaller volume the pressure is larger than the ideal gas equation would predict.

As you may have guessed by its name, the ideal gas equation describes a gas which has a number of properties that real physical gases do not actually have. This type of gas is called an **ideal gas**, and has the following properties:



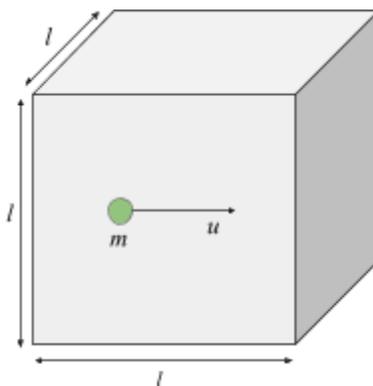
- **No intermolecular forces** act between the particles in the gas. This is equivalent to saying the particles only carry kinetic energy.
- The **size of the particles is negligible** compared to the volume of the gas.
- The collisions between particles are perfectly elastic.
- The motion of the particles is **purely random**.

We can **approximate** a **real gas to behave like an ideal gas** so that we can use the ideal gas law to investigate it. This is a good approximation when the **temperature is large and the pressure is small**. If the pressure is too large, the potential energy associated with the distance between particles will become more significant compared to their kinetic energy.

Deriving the Ideal Gas Law

In the section above we obtained the ideal gas law by an experimental argument. It is also possible to derive the law by modelling the gas using some of the assumptions listed above.

Let's first imagine one particle moving around in a cube with sides of length l . The particle has a mass m moves with a speed u in the direction shown in the diagram.



If the particle collides elastically with the walls of the cube, its **change of momentum** due to each collision will be:

$$\Delta \text{momentum} = mu - (-mu) = 2mu$$

After this collision, the particle will be travelling in the opposite direction as in the diagram. It will then bounce off the opposite side and travel back, corresponding to a distance of $2l$. The time taken for the particle to travel this distance is:

$$t = \frac{\text{distance}}{\text{speed}} = \frac{2l}{u}$$





We know that **force is the rate of change of momentum**, so the force that the wall experiences due to the particle repeatedly colliding with it is:

$$F = \frac{\Delta \text{momentum}}{t} = \frac{2mu}{\frac{2l}{u}} = \frac{mu^2}{l}$$

The **pressure** that the wall experiences is F/A , where $A = l^2$ is the area of the cube side. This gives:

$$\text{Pressure} = p = \frac{F}{A} = \frac{mu^2}{l^3} = \frac{mu^2}{V}$$

In the last step we have used the fact l^3 is the **volume** of the cube. This equation highlights an important point, **the pressure of a gas is caused by collisions between the particles and the container walls.**

So far we have only considered a single particle but we know gases contain many particles. The **total pressure** exerted by the cube of gas will have contributions from each particle in the gas. If we label each particles' speed with a number from 1 to N , we can write the total pressure as:

$$p = \frac{m}{V}(u_1^2 + u_2^2 + \dots + u_N^2)$$

It isn't particularly useful to have to consider the individual speed of each particle. So we define a quantity called the **mean square speed**, which is given as:

$$\bar{u}^2 = \frac{u_1^2 + u_2^2 + \dots + u_N^2}{N}$$

The 'bar' above u^2 indicates that it is an **average quantity**.

Multiplying both sides of this equation by N allows us to replace the sum in the expression for pressure to give:

$$p = \frac{Nm\bar{u}^2}{V}$$

To finish, we need to consider what u actually is. It was the speed of the particle in one particular direction that we chose at the start. Particles in gases don't move in one direction, they move in three! We can then relate the **mean squared speed** to the **mean squared speed in each direction, using the Pythagorean Theorem:**

$$\bar{c}^2 = \bar{u}^2 + \bar{v}^2 + \bar{w}^2$$





Here u , v , w represent the components the velocity in each of the x , y , z directions. If we remember back to our assumptions about the ideal gas, **the particles move randomly**. This means there is no reason for these to be different. Therefore:

$$\bar{u}^2 = \bar{v}^2 = \bar{w}^2 \Rightarrow \bar{u}^2 = \frac{1}{3}\bar{c}^2$$

Substituting this into our equation for pressure gives the **kinetic theory equation**:

$$p = \frac{Nmc^2}{3V} \Rightarrow pV = \frac{Nmc^2}{3}$$

Combining the Ideal Gas and Kinetic Theory Equations

To recap, we have found two equation which describe an ideal gas so far:

We can side of both $pV = k_B NT$ equate to get: $pV = \frac{Nmc^2}{3}$ the right hand

$$\frac{Nmc^2}{3} = Nk_B T$$

If we cancel the N on each side, then multiply by $3/2$ we get the following:

$$\frac{1}{2}mc^2 = \frac{3}{2}k_B T \quad \text{average KE} = \frac{3}{2}k_B T$$

The value on the left is the mean kinetic energy of a particle. **This tells us that the average kinetic energy of a gas particle is proportional to the temperature of the gas.**

This equation tells us the average kinetic energy of a **single particle**. To find the kinetic energy of **one mole** of gas we simply multiply by Avogadro's Number N_A and use the fact $R = N_A k_B$:

$$KE \text{ of one mole} = \frac{3}{2}RT$$

